show

A081296 primes of form $2^K - K$ 1/2

can be derived via bitneg(m, m+1), m even

We restrict $K \geq 3$, all

bitneg(v, w) says to negate v, truncate to w bits

For example,

| decimal | binary | #bits | truncation |
|---|---|---|---|
| 12 | 1100 | 4 | 5 bits |

bitneg $\Rightarrow$ $10011 = \overline{m}$

Note that bitor$(m, \overline{m})$ = all bits set (1)

= one less than a power of 2

let general $m$ = xxx$\phi$ (LSB=$\phi$)

#bits = b

negation gives

$\underbrace{\ldots 111}_{(m+1-b)\text{ set bits}}\underbrace{xxx}_{\overline{m}}1$ (LSB=1) (2)

$$= \overline{m} + \sum_{i=b}^{m} 2^i \quad , m \geq b \qquad (3)$$

using (1) we know

$\overline{m} = (2^b - 1) - m$

→

2/2

(3) then becomes

$$\left(2^b + \dots 2^m\right) + \left(2^b - m - 1\right) \quad (4)$$

Since m is even, m+1 is odd

We must show that (4) can be equivalent to

$2^k - k$

Firstly, note $2^j + 2^j = 2(2^j)$

$= 2^1(2^j) = 2^{(j+1)}$

So a pair of powers of 2 ⇒ next power of 2

e.g. $4+4 = 2^2 + 2^2 = 8 = 2^3$

in (4), $2^b + 2^b$ becomes $2^{b+1}$

We then have $\left(2^{b+1} + \dots 2^m\right) + 2^{b+1} - (m+1) \quad (5)$

Similarly $2^{b+1} + 2^{b+1}$ "rolls up" to $2^{b+2}$ etc

The result becomes

$$2^m + 2^m - (m+1) = 2^{(m+1)} - (m+1) \quad (6)$$

if $k = m+1$, this becomes

$2^k - k$